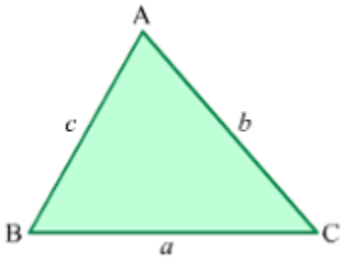
Heron's Formula

- **Perimeter** is the length of the boundary of a closed figure.
- The perimeter of a polygon is the sum of the lengths of all its sides.
  In case of a triangle ABC, with sides of lengths a, b and c units:



  Perimeter of ABC = AB + BC + AC = $a + b + c$
- The **semi-perimeter** of a triangle is half the perimeter of the triangle.

  The semi-perimeter (s) of a triangle with sides a, b and c is $\frac{a+b+c}{2}$.
- The semi-perimeter of a triangle is used for calculating its area when the length of altitude is not known.
- **Area of triangle using Heron's formula:**

  When all the three sides of a triangle are given, its area can be calculated using Heron's formula, which is given by:

  Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

  Here, s is the semi-perimeter of the triangle and is given by, $s = \frac{a+b+c}{2}$

  Example: Find the area of a triangle whose sides are 9 cm, 28 cm and 35 cm.

  Solution: Let a = 9 cm, b = 28 cm and c = 35 cm

  Semi-perimeter, $s = \frac{a+b+c}{2} = \frac{9+28+35}{2}\text{ cm} = 36\text{ cm}$

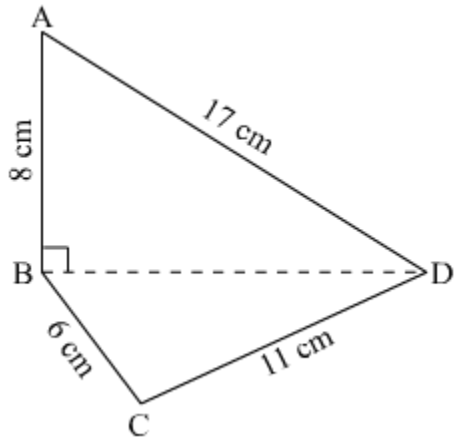
  Area of triangle = $\sqrt{36(36-9)(36-28)(36-35)}\text{ cm}^2$

  $= \sqrt{36 \times 27 \times 8 \times 1}\text{ cm}^2$

  $= 36\sqrt{6}\text{ cm}^2$
- **Area of quadrilaterals using Heron's formula:**

Area of a quadrilateral can also be calculated using Heron's formula. In this, the quadrilateral is divided into two triangles and then the area of each triangle is calculated using Heron's formula.

Example: What is the area of the given quadrilateral?



Solution: ΔABD is a right-angled triangle.

Using Pythagoras Theorem, we get

$$BD = \sqrt{(AD)^2 - (AB)^2} = \left(\sqrt{(17)^2 - (8)^2}\right)\text{cm} = 15 \text{ cm}$$

$$\text{Area } (\Delta ABD) = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times 15 \times 8 = 60\,\text{cm}^2$$

For ΔBCD, let a = 6 cm, b = 11 cm and c = 15 cm

$$\text{Semi-perimeter, } s = \frac{a+b+c}{2} = \left(\frac{6+11+15}{2}\right)\text{cm} = 16 \text{ cm}$$

$$\text{Area } (\Delta BCD) = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{16(16-6)(16-11)(16-15)}\ \text{cm}^2$$

$$= \sqrt{16 \times 10 \times 5 \times 1}\ \text{cm}^2$$

$$= 20\sqrt{2}\ \text{cm}^2$$

$$\text{Area of quadrilateral ABCD} = \left(60 + 20\sqrt{2}\right)\text{cm}^2 = 20\left(3 + \sqrt{2}\right)\ \text{cm}^2$$